Section 11.5 Parametric Equations of Lines

A line L in three-dimensional space is determined by a point $P(x_0, y_0, z_0)$ on L and the direction of L. Thus

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v}$$

is a vector equation of L, where $\mathbf{r_0}$ is the position vector of P (ie, $\mathbf{r_0} =< x_0, y_0, z_0 >$) and $\mathbf{v}$ is any vector parallel to L.

If $\mathbf{v} =< a, b, c >$, we can write

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v}$$

as

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

for $t \in \mathbb{R}$. These equations are called parametric equations of the line L through the point $P(x_0, y_0, z_0)$ and parallel to the vector $\mathbf{v} =< a, b, c >$. Each value of the parameter t gives a point (x, y, z) on L.

Another way of describing a line L is to eliminate the parameter t. If none of a, b, or c is 0, we can solve each parametric equation for t, equate the results, and obtain

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

These equations are called symmetric equations of L.

If one of a, b, or c is 0, we can still eliminate t. For example, if $a = 0$, we could write the equations of L as

$$x = x_0, \quad \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$