Section 11.3 Dot Products

The dot product of two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ is the number

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, $0 \leq \theta \leq \pi$. (ie, θ is the smaller angle between the vectors when they are drawn with the same initial point.) If either $\mathbf{a}$ or $\mathbf{b}$ is $\mathbf{0}$, we define $\mathbf{a} \cdot \mathbf{b} = 0$.

Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are called perpendicular or orthogonal if the angle between them is $\theta = \dfrac{\pi}{2}$. The zero vector $\mathbf{0}$ is considered to be perpendicular to all vectors.

Theorem Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

The Dot Product in Component Form The dot product of $\mathbf{a} =< a_1, a_2, a_3 >$ and $\mathbf{b} =< b_1, b_2, b_3 >$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

Properties of the Dot Product If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$.
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$.
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$.
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$.
5. $\mathbf{0} \cdot \mathbf{a} = 0$.

Projections

Vector Projection of $\mathbf{b}$ onto $\mathbf{a}$:

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \right) \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$$

Scalar Projection of $\mathbf{b}$ onto $\mathbf{a}$:

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$