Section 11.1 Rectangular Coordinates in 3-Space

The direction of the z-axis is determined by the right-hand rule: If you curl the fingers of your right hand around the z-axis in the direction of a 90° counterclockwise rotation from the positive x-axis to the positive y-axis, then your thumb points in the positive direction of the z-axis.

The three coordinate planes divide space into eight parts, called octants. The first octant is determined by the positive axes (ie, $x \geq 0, y \geq 0, z \geq 0$).

If we drop a perpendicular from $P(a, b, c)$ to the xy-plane, we get a point Q with coordinates $(a, b, 0)$ called the projection of P on the xy-plane. Similarly, $R(0, b, c)$ is the projection of P on the yz-plane and $S(a, 0, c)$ is the projection of P on the xz-plane.

$\mathbb{R}^3 = \{(x, y, z) | x, y, z \in \mathbb{R}\}$ is called a three-dimensional rectangular coordinate system.

Distance Formula in Three Dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Equation of a Sphere An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2.$$