Name:___________________________________

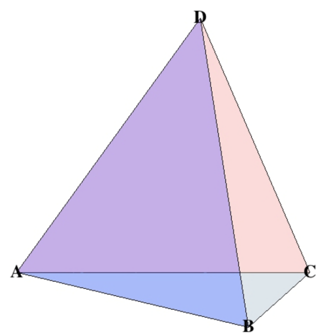
Math 301 Spring 2016

Project - Due 4/25

Choose one of the three projects below to complete. You may work in groups, and each group need only turn in one written report, but please include a summary of each group member's contribution in addition to the written report. Be sure to write complete explanations with each computation. Like starred homework problems, you will be graded on the correctness of the mathematical computations, as well as your written explanations. Have fun!

1. Chapter 11: The Geometry of a Tetrahedron

A tetrahedron is a solid with four vertices, A, B, C, and D, and four triangular faces:



1. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $\mathbf{v}_4$ be vectors with lengths equal to the areas of the faces opposite the vertices A, B, C, and D, respectively, and directions perpendicular to the respective faces and pointing outward. Show that

$$\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 + \mathbf{v}_4 = \mathbf{0}.$$

2. The volume V of a tetrahedron is one-third the distance from a vertex to the opposite face, times the area of that face.

(a) Find a formula for the volume of a tetrahedron in terms of the coordinates of its vertices A, B, C, and D.

(b) Find the volume of the tetrahedron whose vertices are $A(1,1,1), B(1,2,3), C(1,1,2)$, and $D(3,-1,2)$.

3. Suppose the tetrahedron in the figure has a trirectangular vertex B. (This means that the three angles at B are all right angles.) Let P, Q, and R be the areas of the three faces that meet at B, and let S be the area of the opposite face ACD. Using the result of Problem 1, or otherwise, show that

$$S^2 = P^2 + Q^2 + R^2.$$

(This is a three-dimensional version of the Pythagorean Theorem.)

2. Chapter 13: Designing a Dumpster

For this project we locate a trash dumpster in order to study its shape and construction. We then attempt to determine the dimensions of a container of similar design that minimize construction cost.

1. First locate a trash dumpster in your area. Carefully study and describe all details of its construction, and determine its volume. Include a sketch of the container.

2. While maintaining the general shape and method of construction, determine the dimensions such a container of the same volume should have in order to minimize the cost of construction. Use the following assumptions in your analysis:

- The sides, back, and front are to be made from 12-gauge (0.1046 inch thick) steel sheets, which cost \$0.70 per square foot (including any required cuts or bends).
- The base is to be made from a 10-gauge (0.1345 inch thick) steel sheet, which costs \$0.90 per square foot.
- Lids cost approximately \$50.00 each, regardless of dimensions.
- Welding costs approximately \$0.18 per foot for material and labor combined.

Give justification of any further assumptions or simplifications made of the details of construction.

3. Describe how any of your assumptions or simplifications may affect the final result.

4. If you were hired as a consultant on this investigation, what would your conclusions be? Would you recommend altering the design of the dumpster? If so, describe the savings that would result.

3. Chapter 14: Volumes of Hyperspheres

In this project we find formulas for the volume enclosed by a hypersphere in n-dimensional space.

1. Use a double integral and the trigonometric substitution $y = r\sin(\theta)$ to find the area of a circle with radius r.

2. Use a triple integral and trigonometric substitution to find the volume of a sphere with radius r.

3. Use a quadruple integral to find the hypervolume enclosed by the hypersphere $x^2 + y^2 + z^2 + w^2 = r^2$ in $\mathbb{R}^4$. (Use only trigonometric substitution and the reduction formulas for $\displaystyle\int \sin^n(x)dx$ or $\displaystyle\int \cos^n(x)dx$.)

4. Use an n-tuple integral to find the volume enclosed by a hypersphere of radius r in n-dimensional space $\mathbb{R}^n$. (*Hint*: The formulas are different for n even and n odd.)