Name:__________________________________

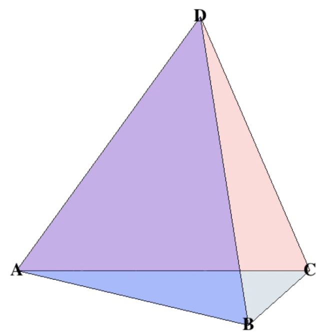
Math 301H Fall 2016

Project - Due 10/5

This project is a portion of the honors component for Math 301H. You may work in groups, and each group need only turn in one written report, but please include a summary of each group member's contribution in addition to the written report. Be sure to write complete explanations with each computation. Like starred homework problems, you will be graded on the correctness of the mathematical computations, as well as your written explanations. Have fun!

The Geometry of a Tetrahedron

A tetrahedron is a solid with four vertices, A, B, C, and D, and four triangular faces:



1. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $\mathbf{v}_4$ be vectors with lengths equal to the areas of the faces opposite the vertices A, B, C, and D, respectively, and directions perpendicular to the respective faces and pointing outward. Show that

$$\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 + \mathbf{v}_4 = \mathbf{0}.$$

2. The volume V of a tetrahedron is one-third the distance from a vertex to the opposite face, times the area of that face.

(a) Find a formula for the volume of a tetrahedron in terms of the coordinates of its vertices A, B, C, and D.

(b) Find the volume of the tetrahedron whose vertices are $A(1, 1, 1), B(1, 2, 3), C(1, 1, 2)$, and $D(3, -1, 2)$.

3. Suppose the tetrahedron in the figure has a trirectangular vertex B. (This means that the three angles at B are all right angles.) Let P, Q, and R be the areas of the three faces that meet at B, and let S be the area of the opposite face ACD. Using the result of Problem 1, or otherwise, show that

$$S^2 = P^2 + Q^2 + R^2.$$

(This is a three-dimensional version of the Pythagorean Theorem.)