Section 3.5 The Cauchy Criterion

A sequence (x_n) is a Cauchy sequence if $\forall \epsilon > 0, \exists$ a natural number $H(\epsilon)$ such that $\forall n, m > H(\epsilon)$,

$$|x_n - x_m| < \epsilon.$$

Lemma If $x_n \to x$, then (x_n) is a Cauchy sequence.

Lemma If (x_n) is Cauchy, then (x_n) is bounded.

Cauchy Convergence Criterion A sequence of real numbers is convergent if and only if it is Cauchy.

(x_n) is contractive if $\exists C, 0 < C < 1$, such that $|x_{n+1} - x_n| \leq C|x_n - x_{n-1}|\ \forall n \in \mathbb{N}$.

Theorem Every contractive sequence is a Cauchy sequence and hence convergent.