Sections 3.5 Homework

1.

a) Let (s_n) be a sequence such that

$$|s_{n+1} - s_n| < 2^{-n} \text{ for all } n \in \mathbb{N}.$$

Prove that (s_n) is a Cauchy sequence and hence a convergent sequence.

b) Is the result in (a) true if we only assume that $|s_{n+1} - s_n| < \dfrac{1}{n}$ for all $n \in \mathbb{R}$?

2. Prove that every subsequence of a Cauchy sequence is Cauchy.

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers, and, for each $n \in \mathbb{N}$, let

$$\begin{aligned} s_n &= a_1 + a_2 + \cdots + a_n, \\ t_n &= |a_1| + |a_2| + \cdots + |a_n|. \end{aligned}$$

Prove that if $\{t_n\}_{n=1}^{\infty}$ is a Cauchy sequence, then so is $\{s_n\}_{n=1}^{\infty}$.