Sections 3.4 Homework

1. Consider the sequences defined as follows:

$$a_n = (-1)^n$$
, $b_n = \frac{1}{n}$, $c_n = n^2$, $d_n = \frac{6n+4}{7n-3}$.

- a) For each sequence, give an example of a monotone subsequence.
- b) For each sequence, give its lim sup and lim inf.
- c) Which of the sequences converges? diverges to ∞ ? diverges to $-\infty$?
- d) Which of the sequences is bounded?
- 2. Prove that $\liminf s_n = -\limsup(-s_n)$.
- 3. Prove that $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.
- 4. Prove that (s_n) is bounded if and only if $\limsup |s_n| < \infty$.
- 5. Let (s_n) be a sequence of nonnegative numbers, and for each n define $\sigma_n = \frac{1}{n}(s_1 + s_2 + \dots + s_n)$.
 - a) Show that

 $\liminf s_n \le \liminf \sigma_n \le \limsup \sigma_n \le \limsup s_n.$

Hint: For the last inequality, show first that M > N implies $\sup\{\sigma_n : n > M\} \le \frac{1}{M}(s_1 + s_2 + \dots + s_N) + \sup\{s_n : n > N\}.$

b) Show that if $\lim s_n$ exists, then $\lim \sigma_n$ exists and $\lim \sigma_n = \lim s_n$.