## Sections 3.4 Homework

1. Consider the sequences defined as follows:

$$
a_{n}=(-1)^{n}, \quad b_{n}=\frac{1}{n}, \quad c_{n}=n^{2}, \quad d_{n}=\frac{6 n+4}{7 n-3}
$$

a) For each sequence, give an example of a monotone subsequence.
b) For each sequence, give its limsup and liminf.
c) Which of the sequences converges? diverges to $\infty$ ? diverges to $-\infty$ ?
d) Which of the sequences is bounded?
2. Prove that $\liminf s_{n}=-\limsup \left(-s_{n}\right)$.
3. Prove that $\limsup \left|s_{n}\right|=0$ if and only if $\lim s_{n}=0$.
4. Prove that $\left(s_{n}\right)$ is bounded if and only if $\lim \sup \left|s_{n}\right|<\infty$.
5. Let $\left(s_{n}\right)$ be a sequence of nonnegative numbers, and for each $n$ define $\sigma_{n}=\frac{1}{n}\left(s_{1}+s_{2}+\cdots+s_{n}\right)$.
a) Show that

$$
\liminf s_{n} \leq \liminf \sigma_{n} \leq \limsup \sigma_{n} \leq \limsup s_{n}
$$

Hint: For the last inequality, show first that $M>N$ implies $\sup \left\{\sigma_{n}: n>M\right\} \leq \frac{1}{M}\left(s_{1}+s_{2}+\cdots+\right.$ $\left.s_{N}\right)+\sup \left\{s_{n}: n>N\right\}$.
b) Show that if $\lim s_{n}$ exists, then $\lim \sigma_{n}$ exists and $\lim \sigma_{n}=\lim s_{n}$.

