Sections 2.3 The Completeness Property of $\mathbb{R}$

Let S be a nonempty subset of $\mathbb{R}$.

a) S is bounded above if there exists $u \in \mathbb{R}$ such that $s \le u$ for all $s \in S$. Each such u is called an upper bound of S.

b) S is bounded below if there exists $w \in \mathbb{R}$ such that $w \le s$ for all $s \in S$. Each such w is called a lower bound of S.

c) A set is bounded if it is both bounded above and below. A set is unbounded if it is not bounded.

Let S be a nonempty subset of $\mathbb{R}$.

a) u is a supremum (least upper bound) of S if
   1) u is an upper bound of S, and
   2) if v is an upper bound of S, then $u \le v$.

b) w is an infimum (greatest lower bound) of S if
   1) w is a lower bound of S, and
   2) if t is any lower bound of S, then $t \le w$.

The Completeness Property of $\mathbb{R}$ Every nonempty set of real numbers that has an upper bound also has a supremum in $\mathbb{R}$.