## Sections 1.1 Sets and Functions

If $A$ and $B$ are nonempty sets, the Cartesian product $A \times B$ of $A$ and $B$ is the set of all ordered pairs $(a, b)$ where $a \in A$ and $b \in B$.

$$
A \times B=\{(a, b): a \in A \text { and } b \in B\}
$$

Let $A$ and $B$ be sets. Then a function from $A$ to $B$ is a set $f$ of ordered pairs in $A \times B$ such that for each $a \in A$ there exists a unique $b \in B$ with $(a, b) \in f$. ie, if $(a, b) \in f$ and $\left(a, b^{\prime}\right) \in f$ then $b=b^{\prime}$.

If $E$ is a subset of $A$, the direct image of $E$ under $f$ is the subset $f(E)$ of $B$ given by

$$
f(E)=\{f(x): x \in E\}
$$

If $H$ is a subset of $B$, then the inverse image of $H$ under $f$ is the subset $f^{-1}(H)$ of $A$ given by

$$
f^{-1}(H)=\{x \in A: f(x) \in H\}
$$

Given propositions $P$ and $Q$, the conjunction of $P$ and $Q$, denoted $P \wedge Q$, is the proposition " P and Q." $P \wedge Q$ is true exactly when both $\overline{P \text { and } Q \text { are true. }}$

Let $f: A \rightarrow B$.
a) $f$ is injective (one-to-one) if $x_{1}, x_{2} \in D(f)$ and $f\left(x_{1}\right)=f\left(x_{2}\right) \Longrightarrow x_{1}=x_{2}$.
b) $f$ is surjective (onto) if $f(A)=B$; ie, if $y \in B$ then there exists $x \in D(f)$ such that $f(x)=y$.
c) If $f$ is injective and surjective, then we say $f$ is bijective.

If $f: A \rightarrow B$ is a bijection, then $g:\{(b, a) \in B \times A:(a, b) \in f\}$ is the inverse of $f$, denoted $f^{-1}$.

