## Sections 1.1 Sets and Functions

If A and B are nonempty sets, the Cartesian product  $A \times B$  of A and B is the set of all ordered pairs (a, b) where  $a \in A$  and  $b \in B$ .

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Let A and B be sets. Then a function from A to B is a set f of ordered pairs in  $A \times B$  such that for each  $a \in A$  there exists a unique  $b \in B$  with  $(a, b) \in f$ . ie, if  $(a, b) \in f$  and  $(a, b') \in f$  then b = b'.

If E is a subset of A, the direct image of E under f is the subset f(E) of B given by

$$f(E) = \{ f(x) : x \in E \}.$$

If H is a subset of B, then the inverse image of H under f is the subset  $f^{-1}(H)$  of A given by

$$f^{-1}(H) = \{x \in A : f(x) \in H\}.$$

Given propositions P and Q, the conjunction of P and Q, denoted  $P \wedge Q$ , is the proposition "P and Q."  $P \wedge Q$  is true exactly when both P and Q are true.

Let  $f : A \to B$ .

- a) f is injective (one-to-one) if  $x_1, x_2 \in D(f)$  and  $f(x_1) = f(x_2) \implies x_1 = x_2$ .
- b) f is surjective (onto) if f(A) = B; ie, if  $y \in B$  then there exists  $x \in D(f)$  such that f(x) = y.
- c) If f is injective and surjective, then we say f is bijective.

If  $f: A \to B$  is a bijection, then  $g: \{(b, a) \in B \times A : (a, b) \in f\}$  is the <u>inverse</u> of f, denoted  $f^{-1}$ .