Sections 1.1 Sets and Functions

If A and B are nonempty sets, the Cartesian product $A \times B$ of A and B is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Let A and B be sets. Then a function from A to B is a set f of ordered pairs in $A \times B$ such that for each $a \in A$ there exists a unique $b \in B$ with $(a, b) \in f$. ie, if $(a, b) \in f$ and $(a, b') \in f$ then $b = b'$.

If E is a subset of A, the direct image of E under f is the subset $f(E)$ of B given by

$$f(E) = \{f(x) : x \in E\}.$$

If H is a subset of B, then the inverse image of H under f is the subset $f^{-1}(H)$ of A given by

$$f^{-1}(H) = \{x \in A : f(x) \in H\}.$$

Given propositions P and Q, the conjunction of P and Q, denoted $P \wedge Q$, is the proposition "P and Q." $P \wedge Q$ is true exactly when both P and Q are true.

Let $f : A \to B$.

a) f is injective (one-to-one) if $x_1, x_2 \in D(f)$ and $f(x_1) = f(x_2) \implies x_1 = x_2$.

b) f is surjective (onto) if $f(A) = B$; ie, if $y \in B$ then there exists $x \in D(f)$ such that $f(x) = y$.

c) If f is injective and surjective, then we say f is bijective.

If $f : A \to B$ is a bijection, then $g : \{(b, a) \in B \times A : (a, b) \in f\}$ is the inverse of f, denoted f^{-1}.