

TEST 3
MATH 310

NAME: _____

November 23, 2015

Each question is worth 10 points. Good luck!

1. Let R be a relation on set A . Prove that $(R^{-1})^{-1} = R$.

2. Prove that $5^n - 2^n$ is divisible by 3 for every natural number n .

3. Let $A = \{ \text{all words in the English language} \}$. Define a relation R on A by xRy if and only if x and y have at least one letter in common. Answer the questions below, justifying each answer with at least a one sentence explanation.

a) Is R reflexive?

b) Is R symmetric?

c) Is R transitive?

4. Let $A = \{a, b, c, d\}$. Give an example of relations $R, S,$ and T on A such that $S \circ R = T \circ R$ but $S \neq T$.

5. Let T be a relation on $\mathbb{R} \times \mathbb{R}$ given by $(x, y)T(a, b)$ iff $x^2 + y^2 = a^2 + b^2$. Prove that T is an equivalence relation. Sketch the equivalence class of $(1, 2)$; of $(4, 0)$.

6. Prove, using the Principle of Mathematical Induction, that

$$\sum_{i=1}^n 2^i = 2^{n+1} - 2 \quad \forall n \in \mathbb{N}.$$