TEST 2
MATH 310

NAME: Key

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Each question is worth 10 points. Good luck!

1. Prove or disprove: Let $a, b, c, d \in \mathbb{Z}$. a divides $b - c$ and a divides $c - d$ implies a divides $b - d$.

Suppose $a | b-c$ and $a | c-d$. Then $\exists\, k, l \in \mathbb{Z}$ s.t. $b-c = ak$ and $c-d = al$. So $b-d = (ak+c) - (c-al) = ak+al = a(k+l)$, where $k+l \in \mathbb{Z}$. Thus, $a | b-d$.

2. Provide either a proof or counterexample for each of these statements:

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x + y = 0$.

Let $x \in \mathbb{R}$ and suppose $y = -x$. Then
$x + y = x - x = 0$.

b) For every positive real number x, there exists a positive real number y less than x with the property that for all positive real numbers z, $yz \geq z$.

Let $x = 1$ and $z = 2$. Then $\forall y < x$, $y \geq 0$, $y < 1$ so $2y < 2$.

3. Prove that if $x \notin B$ and $A \subseteq B$, then $x \notin A$.

Suppose $x \notin B$, $A \subseteq B$, and $x \in A$. Since $x \in A$ and $A \subseteq B$, $x \in B$ ↯.
$\therefore x \notin A$.

4. Let x and y be integers. Prove that if x and y are odd, then $3x - 5y$ is even.

Suppose x and y are odd. Then $\exists\, k, l \in \mathbb{Z}$ s.t. $x = 2k+1$ and $y = 2l+1$. So

$$\begin{aligned} 3x-5y &= 3(2k+1)-5(2l+1) \\ &= 6k+3-10l-5 \\ &= 6k-10l-2 \\ &= 2(3k-5l-1), \text{ where } 3k-5l-1 \in \mathbb{Z}. \end{aligned}$$

Thus, $3x-5y$ is even.

5. For $\mathcal{A} = \{A_n : n \in \mathbb{N}\}$, where $A_n = \left(\frac{1}{n}, 5 - \frac{1}{n}\right]$, find $\bigcup_{A \in \mathcal{A}} A$ and $\bigcap_{A \in \mathcal{A}} A$.

$\bigcup A = (0, 5)$

$\bigcap A = (1, 4]$

6. Prove or disprove: $A \cup (B - C) = (A \cup B) - (A \cup C)$.

Let $A = \{1,2,3\}$, $B = \{4,5\}$, and $C = \{5\}$.

Then

$A \cup (B-C) = \{1,2,3,4\}$ but

$(A \cup B) - (A \cup C) = \{4\}$ so $A \cup (B-C) \neq (A \cup B) - (A \cup C)$.