

TEST 2
MATH 310

NAME: _____

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Each question is worth 10 points. Good luck!

1. Prove or disprove: Let $a, b, c, d \in \mathbb{Z}$. a divides $b - c$ and a divides $c - d$ implies a divides $b - d$.

2. Provide either a proof or counterexample for each of these statements:

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x + y = 0$.

b) For every positive real number x , there exists a positive real number y less than x with the property that for all positive real numbers z , $yz \geq z$.

3. Prove that if $x \notin B$ and $A \subseteq B$, then $x \notin A$.

4. Let x and y be integers. Prove that if x and y are odd, then $3x - 5y$ is even.

5. For $\mathcal{A} = \{A_n : n \in \mathbb{N}\}$, where $A_n = \left(\frac{1}{n}, 5 - \frac{1}{n}\right]$, find $\bigcup_{A \in \mathcal{A}} A$ and $\bigcap_{A \in \mathcal{A}} A$.

6. Prove or disprove: $A \cup (B - C) = (A \cup B) - (A \cup C)$.