

TEST 1
MATH 310

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Each question is worth 10 points. Good luck!

1.

a) Let A and B be sets. Define $A \cup B$.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

b) Define contradiction.

A contradiction is a statement whose truth value is always false.

2. Use truth tables to show the following statements are equivalent:

P	Q	$P \rightarrow Q$	$\sim(P \rightarrow Q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

$\sim(P \rightarrow Q)$ and $P \wedge \sim Q$			
P	Q	$\sim Q$	$P \wedge \sim Q$
T	T	F	F
F	T	F	F
T	F	T	T
F	F	T	F

a) Identify the antecedent (hypothesis) and the consequent (conclusion) for the following conditional sentence.

"A sequence a being bounded is necessary for a sequence a to be convergent."

antecedent: A sequence a is convergent.
consequent: A sequence a is bounded.

b) Write the converse and contrapositive for the conditional sentence in part a).

converse: if a sequence is bounded, then it is convergent.

contrapositive: If a sequence is not bounded, then it is not convergent.

4.

a) Translate the following symbolic sentence into English.

$$\forall x \in \mathbb{N}, x \text{ is odd} \rightarrow x^2 \text{ is odd.}$$

For every natural number x , if x is odd, then x^2 is odd.

b) For the statement in part a), write a useful denial, and give a translation into ordinary English.

$$\exists x \in \mathbb{N} \text{ s.t. } x \text{ is odd} \wedge x^2 \text{ is even.}$$

There exists a natural number x such that x is odd and x^2 is even.

5.

a) If $\sim(P \wedge Q)$ is false, what must be the truth value of Q ?

True

b) If $Q \rightarrow (P \wedge \sim P)$ is true, what must be the truth value of Q ?

False

6. Give an example of nonempty sets A and B such that $A^c \cap B^c \neq (A \cap B)^c$.

$$U = \{1, 2, 3, 4, 5\}$$

$$A = \{1, 2, 3\}$$

$$B = \{3, 4\}$$

$$A^c \cap B^c = \{5\}$$

$$(A \cap B)^c = \{1, 2, 4, 5\}$$