

TEST 1
MATH 310

NAME: Key

September 16, 2015

You may only use the course textbook, the instructor lecture notes, and your class notes. You are not allowed to work together. You are also not allowed to ask anyone questions other than Dr. Kristen Abernathy. By signing your name above, you are pledging the university honor code: "I have neither received unauthorized aid nor given aid on this assignment." Each question is worth 10 points. This is due at the beginning of class on Wednesday, September 23rd. Good luck!

1.

a) Write the following definition using symbolic notation:

"A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous at a if given any positive ϵ , there is a positive δ so that for any real number x , if $|x - a| < \delta$ then $|f(x) - f(a)| < \epsilon$."

b) For the statement in part a), write a useful denial.

2. Give an example of three sets A , B , and C for which

$$A \setminus (B \setminus C) \neq (A \setminus B) \setminus C.$$

3. Determine if the following statements are true or false. Provide a short explanation for each as to why it is true or false.

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x^2 = y^2$.

b) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}, x^2 = y^2$.

c) $\forall x \in \mathbb{R}, \exists ! y \in \mathbb{R}$ such that $x^2 = y^2$.

d) $\forall x \in \mathbb{N}, \exists y \in \mathbb{R}$ such that $x^2 = y^2$.

e) $\forall x \in \mathbb{R}, \exists y \in \mathbb{N}$ such that $x^2 = y^2$.

4. Make Venn diagrams for the sets $(A \cup B) \setminus C$ and $A \cup (B \setminus C)$. What can you conclude about whether one of these sets is necessarily a subset of the other?

1. $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous at a iff $\forall \epsilon > 0, \exists \delta > 0$
s.t. $\forall x \in \mathbb{R}, |x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$.

b) $f: \mathbb{R} \rightarrow \mathbb{R}$ is not continuous at a iff $\exists \epsilon > 0$ s.t.
 $\forall \delta > 0, \exists x \in \mathbb{R}$ s.t. $|x - a| < \delta \wedge |f(x) - f(a)| \geq \epsilon$.

$$2. A = \{1, 2, 3, 4, 5\}$$

$$B = \{2, 3\}$$

$$C = \{4, 5\}$$

$$A - (B - C) = A - \{2, 3\} = \{1, 4, 5\}$$

$$(A - B) - C = \{1, 4, 5\} - C = \{1\}$$

3. a) T: Let $x \in \mathbb{R}$ and choose $y = x$. Then
$$x^2 = y^2.$$

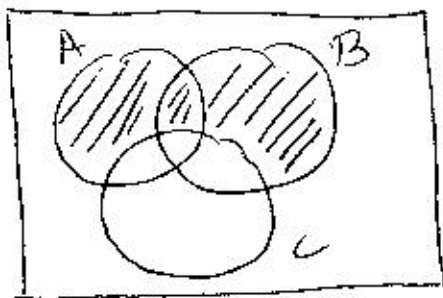
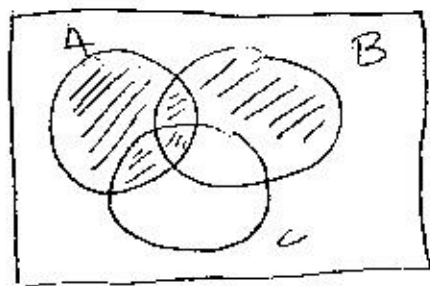
b) F: There is no real number x that works for every real number y .

c) F: Let $x \in \mathbb{R}$ and consider x and $-x$.
If $x \neq 0$, $x^2 = (-x)^2$ but $x \neq -x$.

d) T: Let $x \in \mathbb{N}$ and choose $y = x$. Then
$$x^2 = y^2.$$

e) F: Let $x = 1/2$. There is no natural number y such that $y^2 = 1/4$.

4.

 $(A \cup B) - C$  $A \cup (B - C)$

$$(A \cup B) - C \subseteq A \cup (B - C)$$