Cartesian Products and Relations

Let A and B be sets. R is a relation from A to B iff R is a subset of $A \times B$. If $(a, b) \in R$, we write aRb and say "a is related to b."

The domain of the relation R from A to B is the set $Dom(R) = \{x \in A :$ there exists $y \in B$ such that $xRy\}$.

The range of the relation R is the set $Rng(R) = \{y \in B :$ there exists $x \in A$ such that $xRy\}$.

For any set A, the relation $IA = \{(x, x) : x \in A\}$ is called the identity relation on A.

If R is a relation from A to B, then the inverse of R is the relation $R^{-1} = \{(y, x) : (x, y) \in R\}$.

Let R be a relation from A to B, and let S be a relation from B to C. The composite of R and S is $S \circ R = \{(a, c) :$ there exists $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S\}$.