Section 3.1 Proof Strategies

A theorem is a statement that describes a pattern or relationship among quantities or structures and a proof is a justification of the truth of a theorem. We cannot define all terms nor prove all statements from previous ones. We begin with an initial set of statements, called axioms (or postulates), that are assumed to be true.

Axioms for this class:

1. **Closure properties** The sum of two integers is an integer. The product of two integers is an integer.
2. **Associativity properties** For all $x, y, z \in \mathbb{Z}, x+(y+z) = (x+y)+z$. For all $x, y, z \in \mathbb{Z}, x(yz) = (xy)z$.
3. **Commutativity properties** For all $x, y \in \mathbb{Z}, x + y = y + x$. For all $x, y \in \mathbb{Z}, xy = yx$.
4. **Distributivity properties** For all $x, y, z \in \mathbb{Z}, x(y+z) = xy+xz$. For all $x, y, z \in \mathbb{Z}, (y+z)x = yx+zx$.
5. **Cancellation properties** For all $x, y, z \in \mathbb{Z}$, if $x + z = y + z$, then $x = y$. For all $x, y, z \in \mathbb{Z}, z \neq 0$, if $xz = yz$, then $x = y$.

The direct proof of a statement of the form $P \to Q$ proceeds in a step by step fashion from the antecedent (hypothesis) P to the consequent (conclusion) Q. Since $P \to Q$ is false only when P is true and Q is false, it suffices to show that this situation cannot happen. The direct way to proceed is to assume that P is true and show (deduce) that Q is also true.