EcoSystems, Inc.
1 EcoSystem Dr.
Leseatt, TG 71986

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Math 305 Students
Winthrop University
Rock Hill, SC 29733

March 27, 2014

Dear Differential Equations Students,

As you may know, EcoSystems, Inc., has an immensely successful series of fish farms that dot the scenic Tonwashing coast a short distance from Leseatt and provide a needed and ecologically sound food source for thousands of satisfied customers who are reached daily by our exclusively electric fleet of bright blue delivery vans. As our hallmark has always been the freshness (not to mention the natural and unsullied nature) of our fish, we have to date unfortunately not been able to expand our distribution to include other parts of the country, such, of course, as Rock Hill.

However, we have recently been offered the opportunity to take over a large lake not far from Rock Hill, which would permit the establishment of a fish farm in that location. Needless to say it is essential that if we approach such an undertaking it be from a position of absolute assurance that it will be able to succeed, and it is for the analysis of a model of the farm that we are approaching your class.

It is our experience that the reproduction rate of the fish is both proportional to the size of the fish population and limited by the number of fish that the farm can support. Additionally, especially in such a location as South Carolina, we expect predation to be significant. While it should be possible to restrict this to a reasonable level, predation will produce a measurable effect on the fish population whenever there are significant numbers of fish present.

To model this situation, an outside consultant proposed the model

$$\frac{dN}{dt} = RN\left(1 - \frac{N}{K}\right) - P(1 - e^{-N^2/\epsilon A^2}). \tag{1}$$

Her contract, however, was from several years ago, and required only that she come up with this (and a number of other) model(s) - which we have only recently come to require. Owing to a clerical error, much of the explanation that was associated with this particular model was misplaced, though we understand that N is the number of fish, R, K, P, and A are constants, and ϵ is a parameter in magnitude very much less than 1. A short distance further in the limited documentation that we have on the model, the consultant concludes that "by substituting $t = \alpha\tau$ and $N = \beta u$ into this equation, it is possible to choose α and β to simplify it to the form

$$\frac{du}{d\tau} = ru\left(1 - \frac{u}{q}\right) - (1 - e^{-u^2/\epsilon}), \tag{2}$$

where r and q are, again, constants." In this equation, (i) ϵ is small; (ii) q is close to 1; and (iii) r is related to the reproduction rate of the fish, which we can control through feeding policy - we expect that it may range from $r = 1$ to perhaps $r = 30$.

Your contract states that you are to justify and analyze the model proposed in equation (2), covering in particular the following issues:

> the derivation of equation (2) from equation (1);
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> an analysis of the validity of equation (2) as a model for the fish population in a fish farm;
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> an analysis of whether, based on model equation (2), we may expect a stable fish population from which harvesting could take place; and, if so,
>
> an analysis of how large an initial population of fish will be required to obtain this stable population, and the length of time required for the stable population to be established.

If you should find in the course of your investigation that you have questions regarding this project, you are to contact, as a team, your inimitable instructor, who has agreed to expend copious free time to serve as a mathematical consultant for this project.

Yours sincerely,

"Chuck" R.D. Arwin
President, EcoSystems, Inc.