Math 305

Sections 9.1 Linear Systems with Constant Coefficients

Definition Suppose A is an $n \times n$ matrix. λ is called an eigenvalue of A is there is a nonzero vector $\mathbf{v}$ such that

$$A\mathbf{v} = \lambda\mathbf{v}.$$

Any vector $\mathbf{v}$ satisfying this equation is called an eigenvector associated with the eigenvalue λ.

Theorem Suppose λ is an eigenvalue of A and $\mathbf{v}$ is an associated eigenvector. Then $\mathbf{x}(t) = e^{\lambda t}\mathbf{v}$ is a solution to $\mathbf{x}' = A\mathbf{x}$ and satisfies the initial condition $\mathbf{x}(0) = \mathbf{v}$.

Definition If A is an $n \times n$ matrix, the polynomial

$$p(\lambda) = (-1)^n \det(A - \lambda I) = \det(\lambda I - A)$$

is called the characteristic polynomial of A and

$$p(\lambda) = (-1)^n \det(A - \lambda I) = 0$$

is called the characteristic equation.

Proposition The eigenvalues of a matrix A are the roots of its characteristic polynomial.

Proposition Let A be an $n \times n$ matrix, and let λ be an eigenvalue of A. The set of all eigenvectors associated with λ is equal to the nullspace of $A - \lambda I$. This nullspace is also called the eigenspace of A.