Math 305

Sections 4.5 Inhomogeneous Equations; the Method of Undetermined Coefficients

Theorem Suppose y_p is a particular solution to $y'' + py' + qy = f$ and $y_1,\ y_2$ for a fundamental set of solutions to $y'' + py' + qy = 0$. Then the general solution of $y'' + py' + qy = f$ is

$$y = y_p + c_1 y_1 + c_2 y_2, \text{ or } y = y_p + y_h$$

where y_h is the general solution to the corresponding homogeneous equation $y'' + py' + qy = 0$.

The Method of Undetermined Coefficients It is reasonable to guess that there is a particular solution y_p that is of the same form as f. This method will work for functions with terms that replicate under differentiation.

Forcing Function $f(t)$	Trial Solution $y_p(t)$
e^{rt}	ae^{rt}
$\cos(\omega t)$ or $\sin(\omega t)$	$a\cos(\omega t) + b\sin(\omega t)$
polynomial of degree n	$a_n t^n + a_{n-1}t^{n-1} + \cdots + a_1 t + a_0$

Notes:

1. Each term in $f(t)$ and all of its "new" derivatives must show up in $y_p(t)$.
2. You can also take products and/or sums of the above functions and the form of y_p would be the product/sums of the y_p forms for each function.
3. If any term in the form of y_p repeats a term from the homogeneous solution y_h, then y_p must be multiplied by t or t^2 to eliminate these terms.