

Math 305

Section 2.7 Existence and Uniqueness of Solutions

Existence Theorem Suppose the function $f(t, x)$ is defined and continuous on the rectangle R in the tx -plane. Then given any point $(t_0, x_0) \in R$, the initial value problem

$$x' = f(t, x) \text{ and } x(t_0) = x_0$$

has a solution $x(t)$ defined in an interval containing t_0 . Furthermore, the solution will be defined at least until the solution curve $t \rightarrow (t, x(t))$ leaves the rectangle R .

Uniqueness Theorem Suppose the function $f(t, x)$ and its partial derivative $\frac{\partial f}{\partial x}$ are both continuous on the rectangle R in the tx -plane. Then for any $(t_0, x_0) \in R$, the initial-value problem

$$x' = f(t, x), \quad x(t_0) = x_0$$

has a unique solution.