

Math 305

Section 2.4 Linear Equations

Definition A first-order linear equation is one of the form

$$x' = a(t)x + f(t).$$

If $f(t) = 0$, the equation has the form

$$x' = a(t)x,$$

and the linear equation is said to be homogeneous.

Summary of the method for solving linear equations The equation

$$x' = a(t)x + f(t)$$

can be solved using the following four steps.

1. Rewrite the equation as

$$x' - a(t)x = f(t).$$

2. Multiply by the integrating factor

$$\mu(t) = e^{-\int a(t)dt},$$

so that the equation becomes

$$(\mu x)' = \mu f.$$

3. Integrate this equation to obtain

$$\mu(t)x(t) = \int \mu(t)f(t)dt + C.$$

4. Solve for $x(t)$.