Taylor and Maclaurin Series

Theorem If f has a power series representation at a, that is, if

$$f(x) = \sum n = 0^{\infty} c_n (x-a)^n \text{ for } |x-a| < R,$$

then its coefficients are given by the formula

$$c_n = \frac{f^{(n)}(a)}{n!};$$

ie, if $f(x)$ has a power series expansion at a, then

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

This is called the Taylor series of the function f at a.

When $a = 0$, we have

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots$$

which is a special case of the Taylor series, called the Maclaurin series.