

Section 9.5 Alternating Series, Absolute and Conditional Convergence

Alternating Series An alternating series is a series whose terms are alternating positive and negative.

The Alternating Series Test If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots \quad (b_n > 0)$$

satisfies

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim_{n \rightarrow \infty} b_n = 0$

then the series is convergent.

Absolute Convergence A series $\sum a_n$ is called absolutely convergent if the series of absolute values $\sum |a_n|$ is convergent.

Theorem If a series $\sum a_n$ is absolutely convergent, then it is convergent.

Ratio Test

(i) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and hence convergent).

(ii) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ or $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, the Ratio Test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\sum a_n$.