Section 9.1 <u>Sequences</u>

Definition A sequence is a list of numbers determined by a function a_n, where n denotes positive integers. (ie, the function a_n has domain the positive integers)

Definition A sequence $\{a_n\}$ has the limit L and we write

$$\lim_{n \to \infty} a_n = L \text{ or } a_n \to L \text{ as } n \to \infty$$

if we can make the terms a_n as close to L as we like by taking n sufficiently large. If $\lim_{n \to \infty} a_n$ exists, we say the sequence converges (or is convergent). Otherwise, we say the sequence diverges (or is divergent).

Theorem If $\lim_{x \to \infty} f(x) = L$ and $f(n) = a_n$ when n is a positive integer, then $\lim_{n \to \infty} a_n = L$.

All algebraic limit properties hold for sequences (ie, the limit of the sum is the sum of the limits, etc.)

The Squeeze Theorem If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n \to \infty} a_n = \lim_{n \to \infty} cn = L$, then $\lim_{n \to \infty} bn = L$.

Corollary to Squeeze Theorem If $\lim_{n \to \infty} |a_n| = 0$, then $\lim_{n \to \infty} a_n = 0$.