Slope Fields

Suppose we have a first-order differential equation of the form

$$y' = F(x, y)$$

where F(x, y) is some expression in x and y. If we draw short line segments with slope F(x, y) at several points (x, y), the result is called a slope field (or direction field).

Euler's Method

Euler's method says to start at the point given by the initial condition and proceed in the direction indicated by the direction field. For the general first-order initial-value problem $y' = F(x, y), y(x_0) = y_0$, we use step size h to find the following y-values:

$$y_1(x_1) = y(x_0 + h) = y_0 + hF(x_0, y_0)$$

$$y_2(x_2) = y(x_1 + h) = y_1 + hF(x_1, y_1)$$

$$\vdots$$

$$y_n(x_n) = y(x_{n-1} + h) = y_{n-1} + hF(x_{n-1}, y_{n-1}).$$