Section 7.7 Approximate Integration

Riemann Sums

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i^*)$$

where $\Delta x = \dfrac{b-a}{n}$ and x_i^* are sample points from the interval $[x_{i-1}, x_i]$.

Midpoint Rule

$$\int_a^b f(x)dx \approx M_n = \Delta x[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

where

$$\Delta x = \frac{b-a}{n}$$

and

$$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{ midpoint of } [x_{i-1}, x_i].$$

Trapezoidal Rule

$$\int_a^b f(x)dx \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + \Delta x$.

Error Bounds for Midpoint and Trapezoidal Rules

Suppose $|f''(x)| \leq K$ for $a \leq x \leq b$. If E_T and E_M are the errors in the Trapezoidal and Midpoint Rules, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \text{ and } |E_M| \leq \frac{K(b-a)^3}{24n^2}.$$

Simpson's Rule

$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+4f(x_{n-3})+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$$

where n is even, $\Delta x = \dfrac{b-a}{n}$, and $x_i = a + \Delta x i$.

Error Bound for Simpson's Rule

Suppose that $|f^(4)(x)| \leq K$ for $a \leq x \leq b$. If E_S is the error involved in using Simpson's Rule, then

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}.$$