Math 201 Fall 2011

Review for Test 3

1. Use an appropriate local linear approximation to estimate the value of $\sqrt{24}$.

2. Find: (a) the intervals on which f is increasing, (b) the intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d) the open intervals on which f is concave down, and (e) the x-coordinates of all inflection points.

$f(x) = 5 - 4x - x^2$

$f(x) = \frac{x}{x^2+2}$

$f(x) = x^{2/3} - x$

3. Locate the critical points of

$$f(x) = \frac{x^2}{x^3 + 8}$$

4. Use $f'(x) = 4x^3 - 9x$ to find all critical points of f, and at each critical point determine whether a relative maximum, relative minimum, or neither occurs.

5. Use any method to find the relative extrema of the function f.

(a) $f(x) = x(x-4)^3$

(b) $f(x) = 2x + 3x^{1/3}$

6. Give a graph of the polynomial $f(x) = 2 - x + 2x^2 - x^3$ and label the coordinates of the critical points and inflection points.

7. Find the absolute maximum and minimum values of f on the given closed interval, and state where those values occur.

(a) $f(x) = 8x - x^2$; $[0, 6]$

(b) $f(x) = (x^2 + x)^{2/3}$; $[-2, 3]$

8. Find the absolute maximum and minimum values of f, if any, on the given interval, and state where those values occur.

(a) $f(x) = 3 - 4x - 2x^2$; $(-\infty, \infty)$

(b) $f(x) = \frac{x-2}{x+1}$; $(-1, 5]$

9. The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences along its other two sides. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.

10. A rectangular page is to contain 42 square inches of printable area. The margins at the top and bottom of the page are each 1 inch, one side margin is 1 inch, and the other side margin is 2 inches. What should the dimensions of the page be so that the least amount of paper is used?

11. A church window consisting of a rectangle topped by a semi-circle is to have a perimeter p. Find the radius of the semi-circle if the area of the window is to be maximum.

12. The given equation $x^3 + x - 1 = 0$ has one real solution. Approximate it by Newton's Method.

13. Use Newton's Method to approximate the absolute maximum of $f(x) = x\sin(x)$ on the interval $[0, \pi]$.

14. For $f(x) = x^3 - 3x^2 + 2x$, verify that the hypotheses of Rolle's Theorem are satisfied on $[0, 2]$, and find all values of c in that interval that satisfy the conclusion of the theorem.

15. For $f(x) = x^3 + x - 4$, verify that the hypotheses of the Mean-Value Theorem are satisfied on $[-1, 2]$, and find all values of c in that interval that satisfy the conclusion of the theorem.

16. Determine whether the statement is true or false. Explain your answer.

If f is continuous on a closed interval $[a, b]$ and differentiable on (a, b), then there is a point between a and b at which the instantaneous rate of change of f matches the average rate of change of f over $[a, b]$.

17. Use the fact that

$$\frac{d}{dx}[3x^4 + x^2 - 4x] = 12x^3 + 2x - 4$$

to show that the equation $12x^3 + 2x - 4 = 0$ has at least one solution in the interval $(0, 1)$.