Math 201 Fall 2011

Review for Test 2

1. A function $y = f(x)$ and values of x_0 and x_1 are given.

(a) Find the average rate of change of y with respect to x over the interval $[x_0, x_1]$.

(b) Find the instantaneous rate of change of y with respect to x at the specified value of x_0.

(c) Find the instantaneous rate of change of y with respect to x at an arbitrary value of x_0.

(d) The average rate of change in part (a) is the slope of a certain secant line, and the instantaneous rate of change in part (b) is the slope of a certain tangent line. Sketch the graph of $y = f(x)$ together with those two lines.

$y = x^3; \quad x_0 = 1, \quad x_1 = 2$

2. A function $y = f(x)$ and an x-value x_0 are given.

(a) Find a formula for the slope of the tangent line to the graph of f at a general point $x = x_0$.

(b) Use the formula obtained in part (a) to find the slope of the tangent line for the given value of x_0.

$f(x) = x^2 + 3x + 2; \quad x_0 = 2$

3. Use the definition of the derivative to find dy/dx for the function

$$y = \frac{1}{x+1}.$$

4. Determine whether the statement is true or false. Explain your answer.

(a) If a function f is differentiable at $x = 0$, then f is continuous at $x = 0$.

(b) If $f(x)$ is a cubic polynomial, then $f'(x)$ is a quadratic polynomial.

(c) If $g(x) = f(x)\sin(x)$, then $g'(0) = f(0)$.

5. Show that

$$f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ x + 2 & x > 1 \end{cases}$$

is continuous but not differentiable at $x = 1$. Sketch the graph of f.

6. Find $f'(x)$.

(a) $f(x) = \sqrt{x} + \frac{1}{x}$

(b) $f(x) = (2 - x - 3x^3)(7 + x^5)$

(c) $f(x) = \frac{x-2}{x^4+x+1}$

(d) $f(x) = (2\sqrt{x} + 1)\left(\frac{2-x}{x^2+3x}\right)$

(e) $f(x) = \frac{\sin(x)}{x^2+\sin(x)}$

(f) $f(x) = \frac{1}{(x^5-x+1)^9}$

(g) $f(x) = 4x + 5\sin^4(x)$

7. Find d^2y/dx^2.

(a) $y = 4x^7 - 5x^3 + 2x$

(b) $y = x^2\cos(x) + 4\sin(x)$

(c) $y = \sin(3x^2)$

8. Newton's Law of Universal Gravitation states that the magnitude F of the force exerted by a point with mass M on a point with mass m is

$$F = \frac{GmM}{r^2}$$

where G is a constant and r is the distance between the bodies. Assuming that the points are moving, find a formula for the instantaneous rate of change of F with respect to r.

9. Find $F'(\pi)$ given that $f(\pi) = 10$, $f'(\pi) = -1$, $g(\pi) = -3$, and $g'(\pi) = 2$.

(a) $F(x) = 6f(x) - 5g(x)$

(b) $F(x) = x(f(x) + g(x))$

(c) $F(x) = 2f(x)g(x)$

(d) $F(x) = \frac{f(x)}{4+g(x)}$

10. An Earth-observing satellite can see only a portion of the Earth's surface. The satellite has horizon sensors that can detect the angle θ shown in the accompanying figure. Let r be the radius of the Earth (assumed spherical) and h the distance of the satellite from the Earth's surface.

(a) Show that $h = r(\csc(\theta) - 1)$.

(b) Using $r = 6378$ km, find the rate at which h is changing with respect to θ when $\theta = 30°$. Express the answer in units of kilometers/degree.

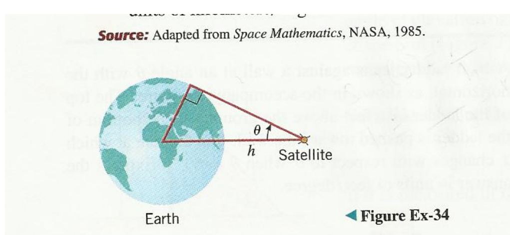


Figure Ex-34

11. Make a conjecture about the derivative by calculating the first few derivatives and observing the resulting pattern.

$$\frac{d^{100}}{dx^{100}}[\cos(x)]$$

12. Given that $f'(x) = \sqrt{3x+4}$ and $g(x) = x^2 - 1$, find $F'(x)$ if $F(x) = f(g(x))$.

13. Find dy/dx by implicit differentiation.

(a) $x^3y^2 - 5x^2y + x = 1$

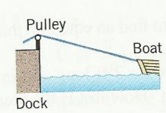
(b) $\cos(xy^2) = y$

14. Use implicit differentiation to find the slope of the tangent line to the curve at the specified point.

$$y^3 + yx^2 + x^2 - 3y^2 = 0; \quad (0, 3)$$

15. Grain pouring from a chute at the rate of 8 ft^3/min forms a conical pile whose height is always twice its radius. How fast is the height of the pile increasing at the instant when the pile is 6 ft high?

16. A boat is pulled into a dock by means of a rope attached to a pulley on the dock (see the accompanying figure). The rope is attached to the bow of the boat at a point 10 ft below the pulley. If the rope is pulled through the pulley at a rate of 20 ft/min, at what rate will the boat be approaching the dock when 125 ft of rope is out?



◀ Figure Ex-30