Math 201 Fall 2011
Review for Test 1

1. Determine whether the statement is true or false. Explain your answer.

(a) If $\lim_{x\to a^+} f(x) = \infty$, then $f(a)$ is undefined.

(b) If $\lim_{x\to a} g(x) = 0$ and $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist (diverges).

2. Sketch a possible graph for a function f with the specified properties.

- the domain of f is $[-2, 1]$
- $f(-2) = f(0) = f(1) = 0$
- $\lim_{x\to -2^+} f(x) = 2$, $\lim_{x\to 0} f(x) = 0$, and $\lim_{x\to 1^-} f(x) = 1$

3. Find the limits.

(a) $\lim_{x\to 3} x^3 - 3x^2 + 9x$

(b) $\lim_{x\to 0} \frac{6x-9}{x^3-12x+3}$

(c) $\lim_{x\to 3^-} \frac{x}{x-3}$

(d) $\lim_{x\to\infty} 2x^3 - 100x + 5$

(e) $\lim_{x\to\infty} \frac{5x^2-4x}{2x^2+3}$

(f) $\lim_{t\to -\infty} \frac{5-2t^3}{t^2+1}$

(g) $\lim_{s\to\infty} \sqrt[3]{\frac{3s^7-4s^5}{2s^7+1}}$

(h) $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right)$

(i) $\lim_{x\to 1} \frac{x^3-x^2}{x-1}$

(j) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x}$

4. Let

$$g(t) = \begin{cases} t-2, & t<0 \\ t^2, & 0 \le t \le 2 \\ 2t, & t>2 \end{cases}$$

Find

$$\text{(a) } \lim_{t\to 0} g(t) \quad \text{(b) } \lim_{t\to 1} g(t) \quad \text{(c) } \lim_{t\to 2} g(t).$$

5. Find a value of the constant k, if possible, that will make the function continuous everywhere.

$$f(x) = \begin{cases} 9-x^2, & x \ge -3 \\ \frac{k}{x^2} & x < -3 \end{cases}$$

6. Show that the equation $x^3 + x^2 - 2x = 1$ has at least one solution in the interval $[-1, 1]$.

7. Use Squeeze Theorem to help calculate the following limit.

$$\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$$