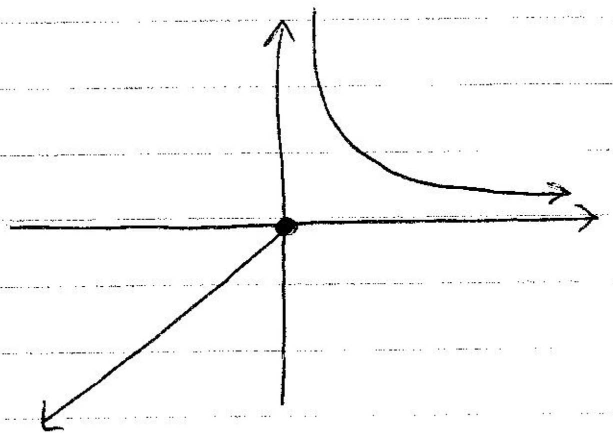
Math 201
Test 1 Review

1. (a) consider the example

$$f(x) = \begin{cases} x & \text{if } x \le 0 \\ 1/x & \text{if } x > 0 \end{cases}$$



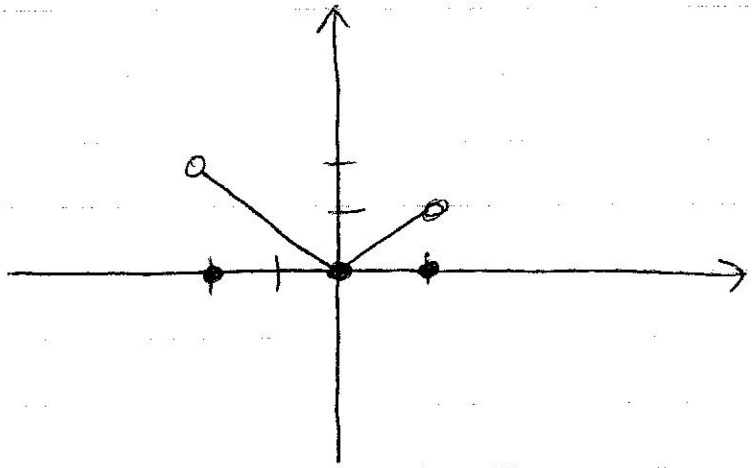
In this example, $\lim_{x \to 0^+} f(x) = \infty$, but $f(0) = 0$. Therefore, this example shows that the statement is false.

(b) Consider the example $f(x) = x^2 - 1$ and $g(x) = x - 1$. Then $\lim_{x \to 1} f(x) = 0$, $\lim_{x \to 1} g(x) = 0$, and

$$\lim_{x \to 1} \frac{f(x)}{g(x)} = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} x + 1 = 2.$$

This example shows that the statement is false.

2.



This is a graph of the function

$$f(x) = \begin{cases} 0 & \text{if } x = -2 \\ -x & \text{if } -2 < x \le 0 \\ x & \text{if } 0 < x < 1 \\ 0 & \text{if } x = 1 \end{cases}$$

3. (a) $\lim\limits_{x \to 3} x^3 - 3x^2 + 9x = (3)^3 - 3(3)^2 + 9(3) = 27$

(b) $\lim\limits_{x \to 0} \dfrac{6x-9}{x^3-12x+3} = \dfrac{6(0)-9}{(0)^3-12(0)+3} = -3$

(c) $\lim\limits_{x \to 3^-} \dfrac{x}{x-3} = -\infty$ (since if we plug-in $x=3$, we get $\frac{\text{nonzero}}{\text{zero}}$ and for values smaller than 3, $\frac{x}{x-3} < 0$)

(d) $\lim\limits_{x \to \infty} 2x^3 - 100x + 5 = \infty$ (degree in numerator is larger than degree in denominator and function is positive for large values of x)

(e) $\lim_{x\to\infty} \frac{5x^2-4x}{2x^2+3} = \frac{5}{2}$ (since degrees in numerator and denominator are equal)

(f) $\lim_{t\to-\infty} \frac{5-2t^3}{t^2+1} = \infty$ (since degree in numerator is larger than degree in denominator and function is positive for large, negative values of t)

(g) $\lim_{s\to\infty} \sqrt[3]{\frac{3s^7-4s^5}{2s^7+1}} = \sqrt[3]{\frac{3}{2}}$ (since degrees in numerator and denominator are equal)

(h) $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) = \sin\left(\lim_{x\to\infty} \frac{1}{x}\right) =$

$\sin(0) = 0$

(i) $\lim_{x\to 1} \frac{x^3-x^2}{x-1} = \lim_{x\to 1} \frac{x^2(x-1)}{x-1} =$

$\lim_{x\to 1} x^2 = 1$

(j) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x} \cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2} =$

$\lim_{x\to 0} \frac{(x^2+4)-4}{x(\sqrt{x^2+4}+2)} = \lim_{x\to 0} \frac{x^2}{x(\sqrt{x^2+4}+2)} =$

$\lim_{x\to 0} \frac{x}{\sqrt{x^2+4}+2} = 0$

4. (a) $\lim_{t \to 0} g(t) = $ DNE since

$\lim_{t \to 0^-} g(t) = \lim_{t \to 0^-} t-2 = -2$ and

$\lim_{t \to 0^+} g(t) = \lim_{t \to 0^+} t^2 = 0$.

(b) $\lim_{t \to 1} g(t) = \lim_{t \to 1} t^2 = 1$

(c) $\lim_{t \to 2} g(t) = 4$ since

$\lim_{t \to 2^-} g(t) = \lim_{t \to 2^-} t^2 = 4$ and

$\lim_{t \to 2^+} g(t) = \lim_{t \to 2^+} 2t = 4$.

5. $f(-3) = 0$

For $f(x)$ to be continuous, we need $\lim_{x \to -3} f(x) = f(-3)$. So, we have

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{k}{x^2} = \frac{k}{9}$, and

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} 9 - x^2 = 0$.

Thus, we need $\frac{k}{9} = 0$ and so we should choose $k = 0$.

6. Let $f(x) = x^3 + x^2 - 2x - 1$. Then
$f(-1) = (-1)^3 + (-1)^2 - 2(-1) - 1 = 1 > 0$ and
$f(1) = (1)^3 + (1)^2 - 2(1) - 1 = -1 < 0$. By
IVT, $\exists$ c in $(-1, 1)$ such that
$f(c) = 0$. So, there exists c in $[-1, 1]$
such that $c^3 + c^2 - 2c - 1 = 0$, ie,

$$c^3 + c^2 - 2c = 1.$$

Thus, c is a solution to the equation
$x^3 + x^2 - 2x = 1$.

7. We know that $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$. This
gives that $-x^4 \leq x^4 \sin\left(\frac{1}{x}\right) \leq x^4$.
Since $\lim\limits_{x \to 0} -x^4 = 0$ and $\lim\limits_{x \to 0} x^4 = 0$, by
squeeze Theorem, $\lim\limits_{x \to 0} x^4 \sin\left(\frac{1}{x}\right) = 0$.