Math 201

Section 5.5 The Definite Integral

$$\int_a^b f(x)dx$$

is the definite integral of $f(x)$ from $x = a$ to $x = b$. a is the upper limit of integration and b is the lower limit of integration.

Theorem If a function f is continuous on an interval $[a, b]$, then f is integrable on $[a, b]$, and the net signed area A between the graph of f and the interval $[a, b]$ is

$$A = \int_a^b f(x)dx.$$

Properties of the Definite Integral

1. $\int_b^a f(x)dx = -\int_a^b f(x)dx$
2. $\int_a^a f(x)dx = 0$
3. $\int_a^b c dx = c(b - a)$, c constant
4. $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
5. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, c constant
6. $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$
7. If $f(x) \geq 0$ for $a \leq x \leq b$, then
$$\int_a^b f(x)dx \geq 0.$$
8. If $f(x) \geq g(x)$ for $a \leq x \leq b$, then
$$\int_a^b f(x)dx \geq \int_a^b g(x)dx.$$
9. If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then
$$m(b - a) \leq \int_a^b f(x)dx \leq M(b - a).$$