Math 201

Section 5.2 The Indefinite Integral

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem If F is an antiderivative of f on an interval I, then the general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

The process of finding antiderivatives is called antidifferentiation or integration. If $F'(x) = f(x)$, then we use the notation

$$\int f(x)dx = F(x) + C.$$

C is called the constant of integration.

Table of Integration Formulas

Suppose $F'(x) = f(x)$ and $G'(x) = g(x)$. Then

1. $\int kf(x)dx = kF(x) + C$ (k is some constant)
2. $\int [f(x) + g(x)]dx = F(x) + G(x) + C$
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
4. $\int \cos(x)dx = \sin(x) + C$
5. $\int \sin(x)dx = -\cos(x) + C$
6. $\int \sec^2(x)dx = \tan(x) + C$
7. $\int \sec(x)\tan(x)dx = \sec(x) + C$
8. $\int \frac{1}{x}dx = \ln(x) + C$
9. $\int e^x dx = e^x + C$

Initial-value problems

$$\frac{dy}{dx} = f(x), \quad y(x_0) = y_0$$

is a special type of a differential equation, called an initial-value problem (IVP). To solve an IVP, first integrate to find an antiderivative for $f(x)$:

$$y = \int f(x)dx = F(x) + C.$$

Then use your initial condition to solve for C :

$$y_0 = y(x_0) = F(x_0) + C$$

which gives

$$C = y_0 - F(x_0).$$