Math 201

Section 4.4 Absolute Maxima and Minima

A function f has an absolute maximum at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f. The number $f(c)$ is called the maximum value of f on D.

Similarly, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the minimum value of f on D.

The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

The Closed Interval Method

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).

2. Find the values of f at the endpoints of the interval.

3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

First Derivative Test for Absolute Extreme Values

Suppose that c is a critical number of a continuous function f defined on an interval.

(a) If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.

(b) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.