Math 201

Section 4.2 Analysis of Functions II: Relative Extrema; Graphing Polynomials

A function f has a local maximum (or relative maximum) at c if $f(c) \geq f(x)$ when x is near c. (This means that $f(c) \geq f(x)$ for all x in some open interval containing c.)

Similarly, f has a local minimum at c if $f(c) \leq f(x)$ when x is near c.

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Fermat's Theorem

If f has a local maximum or minimum at c, then c is a critical number of f.

NOTE: The converse of Fermat's Theorem is not true. That is, if c is a critical number of f, then c might not be a local maximum or minimum.)

The First Derivative Test

Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c (that is, f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.

The Second Derivative Test

Suppose $f''(x)$ is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.