Math 201

Section 3.5 Local Linear Approximation; Differentials

From the definition of derivatives, we have

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

That is, for values of x very close to a,

$$f'(a) \approx \frac{f(x) - f(a)}{x - a}$$

which gives

$$f(x) \approx f'(a)(x - a) + f(a).$$

This is called the linear approximation or tangent line approximation of f at a.

The linear function whose graph is this function, that is,

$$L(x) = f(a) + f'(a)(x - a)$$

is called the linearization of f at a.

Differentials

If $y = f(x)$, where f is a differentiable function, then the differential dx is an independent variable (often viewed as the change in x). The differential dy is then defined in terms of dx by the equation

$$dy = f'(x)dx.$$