Math 201
Section 2.2 The Derivative Function

Definition The function f' defined by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the derivative of f with respect to x. The domain of f' consists of all x in the domain of f for which the limit exists.

Different Ways to Interpret the Derivative

- $f'(a) =$ slope of the line tangent to $f(x)$ at the point $(a, f(a))$.
- $f'(a) =$ instantaneous rate of change of $y = f(x)$ at $x = a$.

Finding an Equation for the Tangent Line to $y = f(x)$ at $x = x_0$
Step 1 Evaluate $f(x_0)$; the point of tangency is $(x_0, f(x_0))$.
Step 2 Find $f'(x)$ and evaluate $f'(x_0)$, which is the slope m of the line.
Step 3 Substitute the value of the slope m and the point $(x_0, f(x_0))$ into the point-slope form of the line

$$y - f(x_0) = f'(x_0)(x - x_0)$$

Definition A function f is differentiable at x_0 if $f'(x_0)$ exists.

Theorem If f is differentiable at x_0, then f is continuous at x_0.
NOTE: The converse of this theorem is not true; that is, there are functions that are continuous but not differentiable.