Math 201

Section 1.3 Limits at Infinity; End Behavior of a Function

(Informal)Definition Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x\to\infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

Note: The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = L \text{ or } \lim_{x\to-\infty} f(x) = L.$$

Theorem Let f be a rational function with numerator $P(x)$ and denominator $Q(x)$. Suppose $deg(P(x)) = n$ and $deg(Q(x)) = m$.

1. If $n > m$, then
$$\lim_{x\to\infty} f(x) = \pm\infty$$
depending on the sign of the leading coefficient of $P(x)$.

2. If $m > n$, then
$$\lim_{x\to\infty} f(x) = 0.$$

3. If $n = m$, then
$$\lim_{x\to\infty} f(x) = \frac{a_n}{b_m}$$
where a_n is the leading coefficient of $P(x)$ and b_m is the leading coefficient of $Q(x)$.