Math 201
Section 1.1 Limits (An Intuitive Approach)

The Tangent Line Problem Given a function and a point on its graph, find an equation of the line that is tangent to the graph at the given point.

The Area Problem Given a function f, find the area between the graph of f and an interval $[a, b]$ on the x-axis.

(Informal)Definition We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$ as x approaches a equals L" if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a.

(Informal)Definition We write

$$\lim_{x \to a^-} f(x) = L$$

and say "the left-hand limit of $f(x)$ as x approaches a is equal to L" if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a and x less than a.
Similarly, if we require that x be greater than a, we get "the right-hand limit of $f(x)$ as x approaches a is equal to L" and we write

$$\lim_{x \to a^+} f(x) = L.$$

(Informal)Definition The notation

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a (on either side of a) but not equal to a.
Note: The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \pm\infty \quad \lim_{x \to a^-} f(x) = \pm\infty \quad \lim_{x \to a^+} f(x) = \pm\infty$$