Math 150

Section 9.3 Applications of Counting

Example 2

In a lottery, 4 numbers are chosen from the numbers 1-30. If your ticket matches: 2 numbers you win \$10, 3 numbers you win \$50, all 4 numbers you win \$1000. You win nothing otherwise. Construct the probability distribution for the lottery and find the expected winnings for a ticket.

Solution

We begin by constructing the probability distribution. To do this, we must first identify our random variable x. A simple way to do this is to see what word follows "expected" in the question. Here, we are asked to find the expected winnings, so x represents the winnings. Since we could win \$1000, \$50, \$10, or \$0, we have:

x	\$1000	\$50	\$10	\$0
P(x)				

The next step is to calculate the probability with each associated value for x.

Let's first consider $P(x = \$1000)$. To win \$1000, the four numbers we choose from 1-30 must match the four numbers on the winning ticket. To find the probability of choosing those four winning numbers, we need to divide how many ways we can choose four out of the four winning numbers with how many ways we can choose any four numbers from 1-30. For the number of ways we can choose four out of the four winning numbers, we have 4C4. For the number of ways we can choose any four numbers from 1-30, we have 30C4. Thus,

$$P(x = \$1000) = \frac{4C4}{30C4} = \frac{1}{27,405}.$$

Now we'll calculate $P(x = \$50)$. To win \$50, three of the numbers we choose from 1-30 must match three numbers on the winning ticket and one number we choose from 1-30 must not match any of the four numbers on the winning ticket. To find this probability, we need to divide how many ways we can choose three out of the four winning numbers and one out of twenty-six non-winning numbers with how many ways we can choose any four numbers from 1-30. For the number of ways we can choose three out of the four winning numbers, we have 4C3. For the number of ways we can choose one out of the twenty-six non-winning numbers, we have 26C1. "and" means multiply, so the number of ways to choose three winning numbers and one non-matching number is (4C3)(26C1). For the number of ways we can choose any four numbers from 1-30, we have 30C4. Thus,

$$P(x = \$50) = \frac{(4C3)(26C1)}{30C4} = \frac{(4)(26)}{27,405} = \frac{104}{27,405}.$$

Next we calculate $P(x = \$10)$. To win \$10, two of the numbers we choose from 1-30 must match two numbers on the winning ticket and two numbers we choose from 1-30 must not match any of the four numbers on the winning ticket. To find this probability, we need to divide how many ways we can choose two out of the four winning numbers and two out of twenty-six non-winning numbers with how many ways we can choose any four numbers from 1-30. For the number of ways we can choose two out of the four winning numbers, we have 4C2. For the number of ways we can choose two out of the twenty-six non-winning numbers, we have 26C2. "and" means multiply, so the number of ways to choose two winning numbers and two non-matching numbers is (4C2)(26C2). For the number of ways we can choose any four numbers from 1-30, we have 30C4. Thus,

$$P(x = \$10) = \frac{(4C2)(26C2)}{30C4} = \frac{(6)(325)}{27,405} = \frac{1950}{27,405}.$$

Finally, we calculate $P(x = \$0)$. The easiest way to do this is to add the probability of winning \$1000, \$50, and \$10 and subtract that sum from 1:

$$1 - \left(\frac{1}{27,405} + \frac{104}{27,405} + \frac{1950}{27,405} \right) = \frac{25,350}{27,405}.$$

Now, we can fill in the probability distribution and calculate the expected winnings:

x	\$1000	\$50	\$10	\$0
P(x)	$\frac{1}{27,405}$	$\frac{104}{27,405}$	$\frac{1950}{27,405}$	$\frac{25,350}{27,405}$
xP(x)	$\frac{1000}{27,405}$	$\frac{5200}{27,405}$	$\frac{19,500}{27,405}$	0

So, expected winnings $= \frac{1000}{27,405} + \frac{5200}{27,405} + \frac{19,500}{27,405} + 0 = \$.94$.

Example 3

DVDs use microchips. A DVD manufacturer rejects a package of 24 microchips if in a sample of 8, at least 1 is defective. If a package of 24 microchips has 3 defective chips, find the probability the package is accepted (not rejected).

Solution

Note that the package will be accepted if the chosen sample of 8 has no defects. To find the probability that the package is accepted, we need to divide how many ways we can choose a sample of 8 with no defects with how many ways we can choose a sample of any 8 chips. Since 21 of the microchips are not defective, the number of ways to choose a sample of 8 microchips with no defects is 21C8 = 203,490. The number of ways to choose a sample of any 8 chips is 24C8 = 735,471. So, the probability that the package is accepted is

$$\frac{21C8}{24C8} = \frac{203,490}{735,471} = .27668.$$