Math 150

Section 9.2 The Multiplication Principle, Permutations, and Combinations

The Multiplication Principle states that if a task can be divided into several parts, then the total number of ways to complete the task is the product of the number of ways to complete each part.

Example 1 A license plate has six characters. The first three characters are letters and the last three characters are numbers. How many different license plates are possible?

Example 2 The United States Postal Service currently uses 5-digit zip codes in most areas. How many zip codes are possible if there are no restrictions on the digits used? How many zip codes are possible if the first digit must be non-zero?

Example 3 How many ways can 5 people be arranged in a line for a picture?

n-Factorial For any natural number n,

$$n! = n(n-1)(n-2)\cdots(3)(2)(1).$$

By definition, $0! = 1$.

Permutations are arrangements of objects when order is important. In general, we talk about having n total objects and want to determine the number of permutations using r of these objects. We use the notation

$$nPr = \frac{n!}{(n-r)!}.$$

Example 4 I have 9 books. How many ways can I arrange 3 of them on a shelf? 6 of them? All 9 of them?

Example 5 What if 4 books are novels, 3 are biographies, and 2 are cookbooks. How many ways can I arrange the books if:

(a) All novels come first?

(b) Cookbooks occupy the first and last spots?

(c) All biographies are first, followed by all novels, followed by the cookbooks?

(d) Each type of book must be together?

Example 6 A club has 25 members. How many ways can a president, vice-president, treasurer, and secretary be elected?

Combinations are sets, subsets, or groups of objects. The order in which the objects appear does NOT matter. In general, we talk about having n total objects and want to determine the number of combinations using r of these objects. We use the notation

$$nCr = \frac{n!}{(n-r)!r!}$$

Look for these words or phrases in problems to determine if it is a permutation or combination problem:

Permutations (order matters)	Combinations (order does not matter)
How many ways can people be lined up?	How many committees are possible?
How many schedules are possible?	How many groups are possible?
How many arrangements are possible?	How many sets are possible?
How many finishing orders are possible?	How many subsets are possible?
How many ways can officers be elected?	

In either case, repetition of objects is not allowed! When repetitions of objects is allowed, you must rely solely on the Multiplication Principle.

Example 7 How many ways can I select or choose 2 of the 9 books above to take on a trip?

Example 8 How many different committees of 4 people can be chosen from the club of 25 people?