**Math 150**

**Section 9.1** Probability Distributions and Expected Value

A random variable is a function that assigns a real number to each outcome of an experiment.

A table that lists all the outcomes with the corresponding probabilities is called a probability distribution.

NOTE: The sum of the probabilities in a probability distribution must always equal 1.

The information in a probability distribution is often displayed graphically as a special kind of bar graph called a histogram.

Example 1 Give the probability distribution for the number of heads showing when three coins are tossed. Draw a histogram for this probability distribution. What would be the probability of tossing no more than 2 heads?

Suppose that the random variable $x$ can take on the $n$ values $x_1, x_2, x_3, \cdots, x_n$. Suppose also that the probabilities that these values occur are, respectively, $p_1, p_2, p_3, \cdots, p_n$. Then the expected value of the random variable is

$$E(x) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \cdots + x_n p_n.$$

Example 2 Find the expected number of heads when tossing three coins.

A game is termed a "fair game" only when the expected net winnings for the game are zero.

| If the Expected Net Winnings for the player are: | Then the game is: |
|---|---|
| Less than zero | Unfair in favor of the operator |
| Zero | A fair game |
| Greater than zero | Unfair in favor of the player |

If the expected net winnings for the player of a game are negative (say -\$1), then the player is paying \$1 too much for each play.

If the expected net winnings for the player of a game are positive (say \$1), then the player is being charged \$1 too little.

Example 3 A player pays \$1 to roll one die. If the player rolls a 1 the "payoff" is \$2. If the person rolls a 3 the payoff is \$1. All other numbers result in a payoff of \$0. Find the expected winnings (or losses) for a person playing this game. Is this a fair game?

Example 4 500 raffle tickets are sold for \$4 each. One ticket holder will win \$600, one ticket holder will win \$200, and two ticket holders will win \$100 each. Find the expected net winnings for a person who buys one ticket.