Math 150

Section 8.5 Conditional Probability and Independent Events

Example 1 Consider the following survey:

	Earned < \$250,000	Earned ≥ \$250,000	Total
University presidents	53	101	154
Community College presidents	21	14	35
Total	74	115	189

Let event A be "earned ≥ \$250,000" and let event B be "university president." Find the following:

- $P(A)$
- $P(B)$
- $P(A \cap B)$
- $P(A|B)$

Conditional Probability

The conditional probability of an event E, given event F, written $P(E \mid F)$ is

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{n(E \cap F)}{n(F)}; \quad P(F) \neq 0.$$

Example 2 Given that $P(E) = .5$, $P(F) = .45$, and $P(E \cup F) = .75$, find $P(E|F)$.

Example 3 Two fair coins were tossed, and it is known that at least one was a head. Find the probability that exactly one coin showed a head.

Product Rule

If $P(E) \neq 0$ and $P(F) \neq 0$, then the definition of conditional probability shows that

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} \text{ and } P(F \mid E) = \frac{P(F \cap E)}{P(E)}.$$

Using the fact that $P(E \cap F) = P(F \cap E)$, we get the product rule:

$$P(E \cap F) = P(E \mid F)P(F) \text{ or } P(E \cap F) = P(F \mid E)P(E).$$

Example 4 Find the probability of drawing a heart on the first draw and a black card on the second if two cards are drawn without replacement from an ordinary deck (52 cards).

Independent Events

Given events E and F, E and F are independent events if

$$P(E \mid F) = P(E) \text{ or } P(F \mid E) = P(F).$$

If events are not independent, they are dependent events.

Product Rule for Independent Events

E and F are independent events if and only if

$$P(E \cap F) = P(E)P(F).$$

Example 5 A medical experiment showed that the probability that a new medicine is effective is .75, the probability that a patient will have a certain side effect is .4, and the probability that both events occur is .3. Decide whether these events are dependent or independent.