Math 150

Section 8.3 Introduction to Probability

A random experiment has outcomes that we cannot predict, but that nonetheless have a regular distribution in a large number of repetitions. We call a repetition a trial. The possible results of each trial are called outcomes.

The sample space (denoted by S) for a random experiment is the set of all possible outcomes.

Example 1 Write sample spaces for the random experiments:

- A day in April is selected for a bicycle race.
- A coin is tossed and a die is rolled.

An event is an outcome, or a set of outcomes, of a random experiment.

Note: An event is a subset of the sample space.

If an event E equals the sample space S, then E is a certain event. If event $E = \varnothing$, then E is an impossible event.

Set Operations for Events

Let E and F be events for a sample space S. Then

$E \cap F$ occurs when both E and F occur;

$E \cup F$ occurs when E or F (or both) occurs;

E' occurs when E does not occur.

Events E and F are disjoint (or mutually exclusive) events if $E \cap F = \varnothing$.

Basic Probability Principle

Let S be a sample space of equally likely outcomes, and let event E be a subset of S. Then the probability that event E occurs is

$$P(E) = \frac{n(E)}{n(S)}.$$

Example 2 If a coin is tossed and a die is rolled, find the probability of each of the given events:

(a) The die shows a 4.

(b) The die shows a number less than 3 and the coin shows a heads.

(c) The coin shows a heads or tails and the die shows a number less than 7.

(d) The die shows a 7.

Example 3 If a coin is tossed and a die is rolled, what is the probability of rolling a 2 or 6?

Example 4

Dismissing class early	Number of occurances
on time	10
1 min early	7
2 min early	1
5 min early	2
10 min early	3
30 min early	1
Total	24

Find the probability of getting out of class:

(a) 5 min early

(b) on time

Properties of Probability

Let S be a sample space consisting of n distinct outcomes $s_1, s_2, \cdots s_n$. An acceptable probability assignment consists of assigning to each outcome s_i a number p_i (the probability of s_i) according to the following rules:

1. The probability of each outcome is a number between 0 and 1:

$$0 \leq p_1 \leq 1,\ \ 0 \leq p_2 \leq 1,\ \ \cdots,\ \ 0 \leq p_n \leq 1.$$

2. The sum of the probabilities of all possible outcomes is 1:

$$p_1 + p_2 + p_3 + \cdots + p_n = 1.$$