Math 150

Section 3.6 Analyzing Arguments with Truth Tables

Testing the Validity of an Argument with a Truth Table

Step 1. Assign a letter to represent each component statement in the argument.

Step 2. Express each premise and the conclusion symbolically.

Step 3. Form the symbolic statement of the entire argument by writing the conjunction of all the premises as the antecedent of a conditional statement, and the conclusion of the argument as the consequent.

Step 4. Complete the truth table for the conditional statement formed in Step 3. If it is a tautology (always true), then the argument is valid; otherwise, it is invalid.

Example 1

If Wake Forest doesn't have a winning season, then Drs. Abernathy will cry.
Wake Forest doesn't have a winning season.

Drs. Abernathy are crying.

Example 2

If she buys another pair of shoes, her closet will overflow.
Her closet overflows.

She bought another pair of shoes.

Example 3 If I've got you under my skin, then you are deep in the heart of me. If you are deep in the heart of me, then you are not really a part of me. You are deep in the heart of me or you are really a part of me. Therefore, if I've got you under my skin, then you are really a part of me.

Solution:

Step 1

Let p represent the statement "I've got you under my skin.", q represent the statement "You are deep in the heart of me.", and r represent the statement "You are really a part of me."

Step 2

Above becomes

$$\begin{array}{l} p \to q \\ q \to \sim r \\ q \vee r \\ \hline p \to r \end{array}$$

Step 3

$[(p \to q) \wedge (q \to \sim r) \wedge (q \vee r)] \to (p \to r)$

Step 4

p	q	r	$\sim r$	$p \rightarrow q$	$q \rightarrow \sim r$	$q \vee r$	$(p \rightarrow q) \wedge (q \rightarrow \sim r) \wedge (q \vee r)$	$p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow \sim r) \wedge (q \vee r)] \rightarrow (p \rightarrow r)$
T	T	T	F	T	F	T	F	T	T
T	T	F	T	T	T	T	T	F	**F**
T	F	T	F	F	T	T	F	T	T
T	F	F	T	F	T	F	F	F	T
F	T	T	F	T	F	T	F	T	T
F	T	F	T	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F	T	T

The presence of at least one F in the final column makes this argument invalid.