Math 150

Section 3.4 More on the Conditional

Notation	Type of Statement	Example
$p \to q$	conditional	"If $x = 2$, then $x^2 = 4$."
$q \to p$	converse	"If $x^2 = 4$, then $x = 2$."
$\sim p \to \sim q$	inverse	"If $x \neq 2$, then $x^2 \neq 4$."
$\sim q \to \sim p$	contrapositive	"If $x^2 \neq 4$, then $x \neq 2$."

Truth Table

p	q	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Note: $p \to q \equiv \sim q \to \sim p$ and $q \to p \equiv \sim p \to \sim q$

Common Translations of $p \to q$

If p, then q.	p is sufficient for q.
If p, q.	q is necessary for p.
p implies q.	All p are q.
q if p.	p only if q.

Example 1 Write each statement in the form "if p, then q."

"Being in Rock Hill is sufficient for being in South Carolina."

"Being an environmentalist is necessary for being elected."

"The principal will hire more teachers only if the school board approves."

Definition The biconditional is a compound statement of the form "p if and only if q."

Notation: p iff q or $p \leftrightarrow q$

Truth Table for $p \leftrightarrow q$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q (\equiv p \to q \wedge q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T