

## MATH 150 Lecture Notes

### How to Setup a Truth Table

The following is a technique that can be used to systematically setup all of the possible combinations of truth values that are placed in the first columns of truth tables.

Step 1: Identify the number of distinct letters in the compound statement and place the letters in alphabetical order across the top of the truth table.

Step 2: Beginning with the letter to the furthest right (last alphabetically), alternate from T to F down its column for as many rows as your truth table has (recall that having  $n$  distinct letters means that your table will have  $2^n$  rows).

Step 3: Move one letter to the left, and alternate from T's to F's down its column every two rows.

Step 4: Move another letter to the left, and alternate from T's to F's down its column every four rows.

Step 5: Repeat this process for each letter (i.e. with each move to the left, double the number of rows it takes to alternate from T's to F's and back).

Let's illustrate with a couple of examples.

Ex: Setup a truth table for the compound statement  $(p \rightarrow q) \wedge \sim p$ .

Step 1: We have two distinct letters,  $p$  and  $q$ , so we place them alphabetically in the truth table:

p	q
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Step 2: We expect  $2^2 = 4$  rows, so we alternate T's and F's down the  $q$  column 4 times:

p	q
	T
	F
	T
	F

Step 3: Move one column to the left (the  $p$  column), and this time alternate T's and F's every 2 rows:

p	q
T	T
T	F
F	T
F	F

Our truth table is now filled in with all possible combinations of truth values for  $p$  and  $q$ , and we are ready to begin adding columns to the right of our letters to build toward the compound statement of interest.

Ex: Setup a truth table for the compound statement  $(\sim r \vee q) \rightarrow (\sim s \wedge \sim p)$ .

Step 1: We have four distinct letters,  $p$ ,  $q$ ,  $r$ , and  $s$ , so we place them alphabetically in the truth table:

p	q	r	s
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Step 2: We expect  $2^4 = 16$  rows, so we alternate T's and F's down the  $s$  column 16 times:

p	q	r	s
			T
			F
			T
			F
			T
			F
			T
			F
			T
			F
			T
			F
			T
			F
			T
			F

Step 3: Move one column to the left (the  $r$  column), and this time alternate T's and F's every 2 rows:

p	q	r	s
		T	T
		T	F
		F	T
		F	F
		T	T
		T	F
		F	T
		F	F
		T	T
		T	F
		F	T
		F	F
		T	T
		T	F
		F	T
		F	F

Step 4: Move one column to the left (the  $q$  column), and this time alternate T's and F's every 4 rows:

p	q	r	s
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

Step 5: Finally, move to the left again (to the  $p$  column), and alternate T's and F's every 8 rows to complete the setup:

p	q	r	s
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F