## Differential Equations Seminar: Week 6 Exercises

1. Show that the system $\begin{aligned} & x^{\prime}=-4 x-y+x\left(x^{2}+y^{2}\right) \\ & y^{\prime}=x-4 y+y\left(x^{2}+y^{2}\right)\end{aligned}$ has a nontrivial periodic solution.
2. Show that the system $\begin{aligned} x^{\prime} & =-x-y+x\left(3 x^{2}+y^{2}\right) \\ y^{\prime} & =x-y+y\left(3 x^{2}+y^{2}\right)\end{aligned}$ has a limit cycle.
3. Show that the system $\begin{aligned} & x^{\prime}=x-y-x\left(x^{2}+\frac{3}{2} y^{2}\right) \\ & y^{\prime}=x+y-y\left(x^{2}+\frac{3}{2} y^{2}\right)\end{aligned}$ has a limit cycle.
4. Apply the Poincaré-Bendixson Theorem to the system $\quad \begin{aligned} & x^{\prime}=3 x-y-x e^{x^{2}+y^{2}} \\ & y^{\prime}=x+3 y-y e^{x^{2}+y^{2}}\end{aligned}$.
5. Apply the Poincaré-Bendixson Theorem to the second-order equation $y^{\prime \prime}-y^{\prime}\left(1-y^{\prime 2}\right)+y=0$. (Hint: Convert the equation into a system of first-order equations, draw the nullclines, and try to construct a trapping region.)
