## **Differential Equations Seminar: Week 6 Exercises**

1. Show that the system  $\begin{array}{ll} x' &= -4x-y+x(x^2+y^2)\\ y' &= x-4y+y(x^2+y^2) \end{array}$  has a nontrivial periodic solution.

2. Show that the system  $\begin{array}{ll} x' &= -x-y+x(3x^2+y^2)\\ y' &= x-y+y(3x^2+y^2) \end{array}$  has a limit cycle.

3. Show that the system  $\begin{array}{cc} x' &= x-y-x(x^2+\frac{3}{2}y^2)\\ y' &= x+y-y(x^2+\frac{3}{2}y^2) \end{array}$  has a limit cycle.

4. Apply the Poincaré-Bendixson Theorem to the system  $\begin{array}{ll} x' &= 3x-y-xe^{x^2+y^2}\\ y' &= x+3y-ye^{x^2+y^2} \end{array}.$ 

5. Apply the Poincaré-Bendixson Theorem to the second-order equation  $y'' - y'(1 - y'^2) + y = 0$ . (Hint: Convert the equation into a system of first-order equations, draw the nullclines, and try to construct a trapping region.)