Differential Equations Seminar: Week 1 Exercises

1. Verify that $x(t) = Ce^{\frac{t^2}{2}}$ is a solution of x' = tx.

2. Verify that $\mathbf{x}(t) = \begin{pmatrix} \sin(2t) \\ \cos(2t) \end{pmatrix}$ is a solution of $\mathbf{x}' = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \mathbf{x}$.

3. Determine whether each of the following systems is linear/nonlinear. For those that are linear, determine which systems are homogeneous/inhomogeneous:

(a)
$$\begin{aligned} x_1' &= -x_2 \\ x_2' &= -x_1 - 2x_2 + 5\sin(t) \\ x_1' &= -2x_1 + x_1x_2 \end{aligned}$$

(b)
$$\begin{array}{c} x_1 \\ x_2' \\ x_2' \end{array} = -3x_1 - x_2 \end{array}$$

(c)
$$\begin{array}{cc} x_1' &= x_1 + (\sin t)x_2 \\ x_2' &= 2tx_1 - x_2 \end{array}$$

(d)
$$\begin{array}{cc} x'_1 &= x_1 + t \sin(x_2) \\ x'_2 &= 2tx_1 - x_2 \end{array}$$

4. Convert each of the following differential equations to a system of first-order equations using matrix-vector notation:

(a)
$$y'' + 2y' + 4y = 0$$

(b) $x''' + xx'' = \cos(t)$